O K L A H O M A   S T A T E   U N I V E R S I T Y

SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING

ECEN 5513
Stochastic Systems
Fall 2007
Final Exam

PLEASE DO ALL FIVE PROBLEMS

Name : ______________________________

E-Mail Address:_______________________________________

Problem 1:

Suppose height to the bottom of clouds is a Gaussian random variable X for which $a_X = 4000\text{m}$ and $\sigma_X = 1000\text{m}$. A person bets that cloud height tomorrow will fall in the set $A = \{1000\text{m} < X \le 3300\text{m}\}$ while a second person bets that height will be satisfied by $B = \{2000\text{m} < X \le 4200\text{m}\}$. A third person bets they are both correct. Find the probabilities that each person will win the bet.

Problem 2:

A certain "soft" limiter accepts a random input voltage *X* and limits the amplitudes of an output random variable *Y* according to

$$Y = \begin{cases} V(1-e^{-X/a}), & X \geq 0 \\ -V(1-e^{X/a}), & X < 0 \end{cases},$$

where $V > 0$ and $a > 0$ are constants. Show that the probability density of *Y* is

$$f_Y(y) = \frac{a}{(V-y)} f_X\left[a\ln\left(\frac{V}{V-y}\right)\right]u(y) + \frac{a}{(V+y)} f_X\left[-a\ln\left(\frac{V}{V+y}\right)\right]u(-y)$$

where $f_X(x)$ is the probability density of *X*.

<u>Problem 3</u>:

Given two random variables X and Y, find the probability density function of the random variable

$$Z = \frac{\min(X, Y)}{\max(X, Y)}$$

in terms of $f_X(x)$ and $f_Y(y)$.

Problem 4:

Given $W = (aX + 3Y)^2$ where X and Y are zero-mean random variables with variances $\sigma_X^2 = 4$ and $\sigma_Y^2 = 16$. Their correlation coefficient is $\rho = -0.5$.

a) Find a value for the parameter a that minimizes the mean value of *W*.
b) Find the minimum mean value.

Problem 5:

Given two random processes $X(t)$ and $Y(t)$. Find expressions for autocorrelation function of $W(t) = X(t) + Y(t)$ if

a) $X(t)$ and $Y(t)$ are correlated.

b) They are uncorrelated.

c) Thet are uncorrelated with zero-means.